The Economics of OTC markets

Lecture 2

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Bachelier Lectures

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Dealer profits

- The *steady-state* flow of profits generated by the dealer sector in the semicentralized market

$$\Pi(\lambda) := \lambda \int_{\mathcal{D}} \theta \left(P - R(\delta)\right) \left(\mathbf{1}_{\{\delta < \delta^\star\}} d\Phi_1^{\lambda}(\delta) - \mathbf{1}_{\{\delta > \delta^\star\}} d\Phi_0^{\lambda}(\delta)\right)$$
$$= \frac{\lambda\theta \left|L(s, \gamma; F)\right|}{(\gamma + \lambda)\left(r + \gamma + \lambda(1 - \theta)\right)}$$

attains its maximum at

$$\lambda^* := \left(\gamma \times \frac{r + \gamma}{1 - \theta}\right)^{1/2}$$

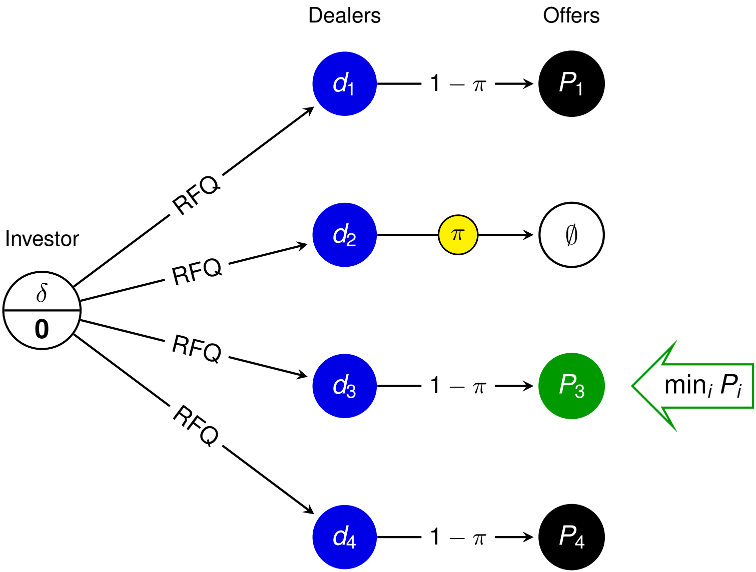
$\Rightarrow$ The optimal "monopolistic" contact rate is *finite* and increasing with respect to γ, r, and θ

Request for quotes

An alternative price-setting mechanism

- Investors contact dealers at rate λ
- Can send a request for quotes to $n \geq 2$ randomly selected dealers
- Each dealer responds with probability $1 - \pi \in (0, 1)$
- Data for CDS market: $\pi \approx 10\%$
- Dealers participate in a small auction as in the price dispersion paper of Burdett and Judd (83)
- Dealers have complete infomation about investors types
- Focus wlog on a buyer sending an RFQ

RFQ with 4 dealers



A dealer's profits

- Denote by P the interdealer price
- Other respondent dealers each draw a θ_i from a distribution G on $[0, 1]$ and quote $(1 - \theta_i)\, P + \theta_i R(\delta)$

$\Rightarrow$ A dealer quoting θ earns

$$\Pi(\theta) := \theta\,(R(\delta) - P) \sum_{k=1}^{n} \phi_k\,(1 - G(\theta-))^{k-1}\,\alpha_k(\theta)$$

where ϕ_k is the probability of $k - 1$ other quotes, and

$$\alpha_k(\theta) := \sum_{\ell=0}^{k-1} \frac{C_\ell^{k-1}}{1+\ell} \left[\frac{\Delta G(\theta)}{1 - G(\theta-)}\right]^\ell \left[1 - \frac{\Delta G(\theta)}{1 - G(\theta-)}\right]^{k-1-\ell}$$

gives the probability that θ is accepted conditional on all other offers being *weakly* dominated

Derivation

- Probability of k quotes: $\phi_k := C_{k-1}^{n-1} (1-\pi)^{k-1} \pi^{n-k}$
- Probability that $k-1$ other quotes are *weakly* dominated:

$$\mathbf{P}\left[\theta \leq \min_{i \leq k-1} \theta_i | \theta\right] = (1 - G(\theta-))^{k-1}$$

- A weakly dominated quote is equal to θ with probability $1-p$ and strictly greater with probability

$$p := \mathbf{P}\left[\theta < \theta_i | \theta \text{ and } \theta \leq \theta_i\right] = 1 - \frac{\Delta G(\theta)}{1 - G(\theta-)}$$

- If there are ℓ quotes equal to θ then one is accepted at random and the dealer wins with probability $1/(1+\ell)$

Equilibrium

- An *equilibrium* is a distribution such that

$$\Pi(\theta') \leq \sup_{q \in \text{supp}(G)} \Pi(q) = \Pi(\theta), \qquad \forall (\theta', \theta) \in [0,1] \times \text{supp}(G)$$

In words: Dealers should be indifferent to any $\theta \in \text{supp}(G)$ and should have no incentives to quote outside of that set

R1 If $\psi_1 < 1$ where

$$\psi_k := \mathbf{P}\left[k \text{ quotes} | \ell \geq 1 \text{quotes}\right] = \frac{C_k^n \left(1-\pi\right)^k \pi^{n-k}}{1-\pi^n}$$

then either $G = \text{dirac}_0$ or G is continuous on $[0,1]$ and supported on $[\underline{\theta}, 1]$ for some $\underline{\theta} \in (0,1)$

Proof of R1

Assume $\psi_1 < 1$ and let $\sigma = \text{supp}(G)$

1. If G is not concentrated at 0
   $\Rightarrow \exists \theta > 0$ such that $G(\theta -) < 1 \Rightarrow \overline{\Pi} = \Pi(\theta) > 0$
   $\Rightarrow \underline{\theta} = \inf\{\sigma\} > 0$ because $\Pi(0) < \overline{\Pi}$
2. If $\overline{\theta} = \max\{\sigma\} < 1$ then offering $\overline{\theta} + \epsilon$ is a profitable deviation as it improves the terms of trade by a discrete amount but only reduce the trade probability by an infinitesimal amount
3. If G include a point mass at some $\theta_1 \in \sigma$ then quoting $\theta_1 - \epsilon$ is a proditable deviation because it improves the trade probability by a discrete amount by eliminiating all offers at θ_1, but only worsens the terms of trade by an infinitesimal amount

Proof of R1

Assume $\psi_1 < 1$ and let $\sigma = \text{supp}(G)$

5. If G is flat over some interval $[\theta_1, \theta_2] \subseteq \sigma$ then quoting $\theta_2 \notin \sigma$ is a profitable deviation as it decreases the dealer's probability of trade by an arbitrary small amount but imporoves the terms of trade by a discrete amount

Equilibrium

R2 In the *unique* equilibrium of the game:

1. If $\psi_1 = 0$ then $G = \text{dirac}_0$ (Bertand)
2. If $\psi_1 = 1$ then $G = \text{dirac}_1$ (Monopoly)
3. If $\psi_1 \in (0, 1)$ then

$$G(\theta) = \mathbf{1}_{\{\theta > \phi_1\}} \frac{1 - (\theta/\phi_1)^{\frac{1}{1-n}}}{1 - \pi}$$

R3 The average transaction price of any investor is the same as in a semicentralized market with $\theta \equiv \psi_1$

R4 The implied bargaining power ψ_1 is decreasing in the number n of contacted dealers and in the probability $1 - \pi$ that a contacted dealer quotes a price

Proof of R2

Assume that the probability $\psi_1 \in (0, 1)$

1. $G = \text{dirac}_0$ cannot be an equilibrium: If it was then any dealer would prefer to quote $\theta > 0$ because $\Pi(\theta) > 0$ due to the fact that $\psi_1 > 0$ guarantees a strictly positive trade probability

$\Rightarrow$ R1: G is continuous with support $[\underline{\theta}, 1]$ for some $\underline{\theta} > 0$

2. This implies that $\Pi(\theta) = \Pi(1)$ for all θ in that interval. Expanding this equality and using that $\alpha_k(\theta; G) = 1$ when G is continuous we deduce that

$$\frac{\Pi(1) - \Pi(\theta)}{R(\delta) - P} = \phi_1 - \sum_{k=1}^{n} \phi_k \theta \left(1 - G(\theta)\right)^{k-1} = 0, \quad \theta \in [\underline{\theta}, 1]$$

and the result follows by solving this equation

Transaction price

- In equilibrium:

$$H(\theta) := \mathbf{P}\,[\text{best quote} \leq \theta] = \sum_{k=1}^{n} \psi_k \left(1 - (1 - G(\theta))^k\right)$$

$\Rightarrow$ The *expected* buying price of the investor is

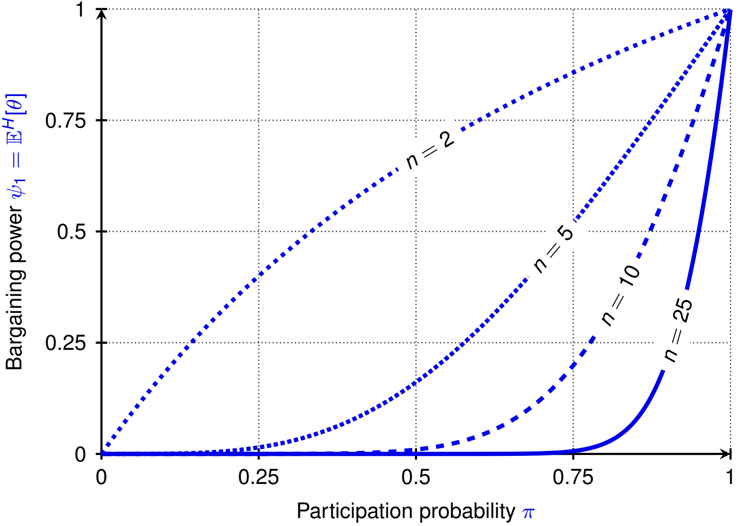
$$\theta^* R(\delta) + (1 - \theta^*)\, P$$

with the bargaining power

$$\theta^* := \mathbb{E}^H[\theta] = \int_0^1 \sum_{k=1}^{n} k\psi_k \theta\,(1 - G(\theta))^{k-1}\, dG(\theta) \equiv \psi_1$$

where the last equality follows by computing the inner sum and then integrating the result

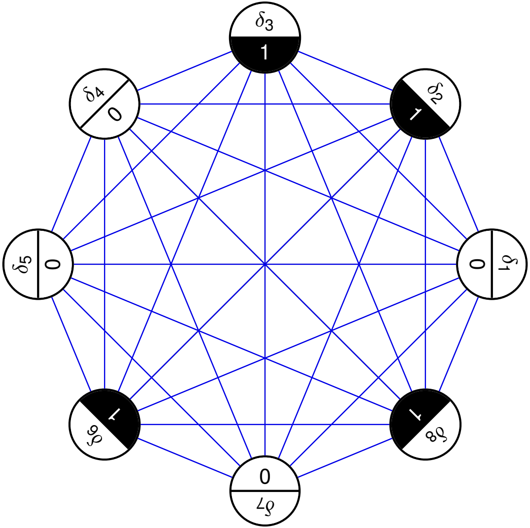
Equilibrium



Decentralized market

- Bilateral trade
  - Contact at rate λ
  - Nash bargaining under complete information
  - Buyer bargaining power $\theta_0 \in [0, 1]$
- Key novelty:
  - (Φ_{0t}, Φ_{1t}) depend on trading decisions
  - Trading decisions depend on value of search options
  - Value of search options depend on (Φ_{0t}, Φ_{1t})
- Serves as a building block for a model of frictional intermediation where investors trade through dealers (SC) and dealers trade in a frictional market

Decentralized market



Reservation values

- Proceeding as in the semicentralized market setting shows that reservation values solve

$$
\begin{aligned}
rR_t(\delta) = \dot{R}_t(\delta) + \delta + \gamma\mathbb{E}^F\left[R_t(x) - R_t(\delta)\right] \qquad \text{(RV)} \\
\text{(resell)} \quad + \lambda\theta_1 \int_{\mathcal{D}} \left(R_t(x) - R_t(\delta)\right)^+ d\Phi_{0t}(x) \\
\text{(buy-back)} \quad - \lambda\theta_0 \int_{\mathcal{D}} \left(R_t(\delta) - R_t(x)\right)^+ d\Phi_{1t}(x)
\end{aligned}
$$

- RVs depend on the distributions $\Rightarrow$ on time!
- Equation (RV) admits a *unique solution* such that $e^{-rT}R_T(\delta) \to 0$ This solution is *strictly increasing in δ*, bounded, and absolutely continuous in time and type

Distributions

- $R_t(\delta)$ is strictly increasing in δ

$\Rightarrow$ An owner at δ_1 *always* sells to a nonowner at $\delta_0 > \delta_1$

$\Rightarrow$ The distributions of types among owners and nonowners can be solved for independently of RVs!

- The distribution of types among owners solves

$$\dot{\Phi}_{1t}(\delta) = \gamma\left(sF(\delta) - \Phi_{1t}(\delta)\right) + \int_{\mathcal{D}\times\mathcal{D}} \lambda\left(\overbrace{\mathbf{1}_{\{x\le y\le\delta\}} - \mathbf{1}_{\{x\le\delta\wedge y\}}}^{\{x\le\delta\le y\}}\right) d\Phi_{1t}(x)d\Phi_{0t}(y) = \gamma\left(sF - \Phi_{1t}\right) + \lambda\Phi_{1t}\left(1 - s - \Phi_{0t}\right)$$

- Quadratic DE since $\Phi_{0t} + \Phi_{1t} = F$

Distributions

- Recall $F_0 = F_t = F$
- The equilibrium distribution of types among owners of the asset is explicitly given by

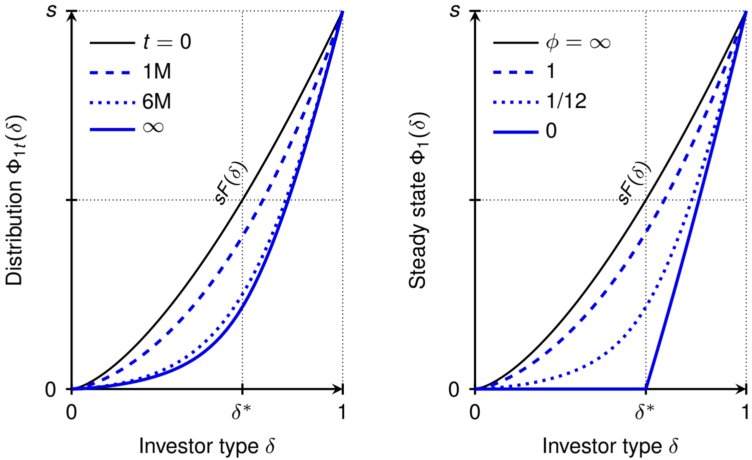
$$\Phi_{1t} = \Phi_1 + \frac{(\Phi_{10} - \Phi_1)\,\Gamma}{\Gamma + (\Phi_{10} - \Phi_1 - \Gamma)\,(e^{\lambda\Gamma t} - 1)}$$

and converges to the **steady state**

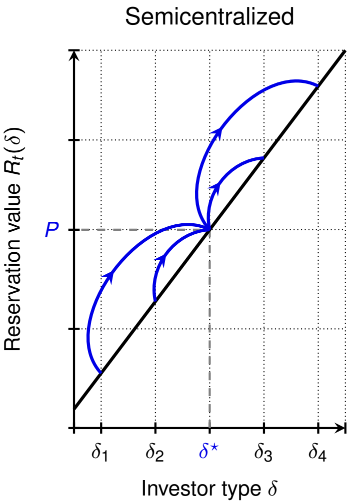
$$\begin{aligned}\Phi_1 &:= \frac{1}{2}\Gamma - \frac{1}{2}(1 - s + \phi - F) \\ &= \frac{1}{2}\left((1 - s + \phi - F)^2 + 4s\phi F\right)^{1/2} - \frac{1}{2}(1 - s + \phi - F)\end{aligned}$$

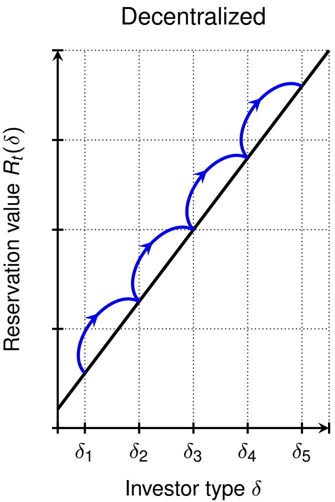
with the constant $\phi := \gamma/\lambda$

Distributions

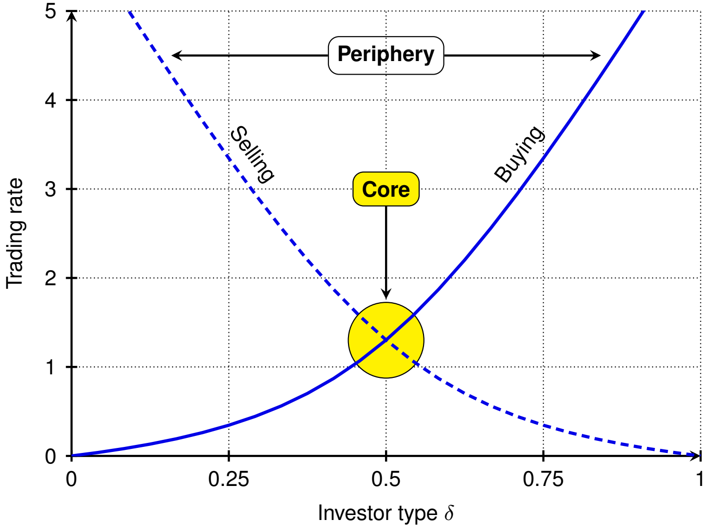


Trading patterns



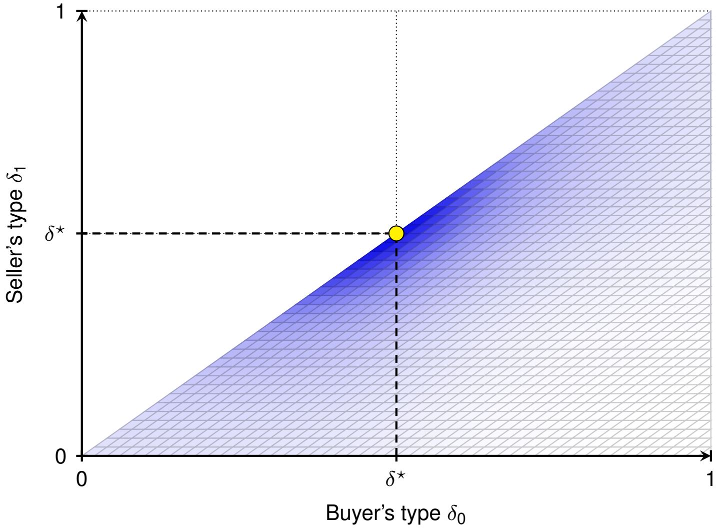


Trading rate



Trading network

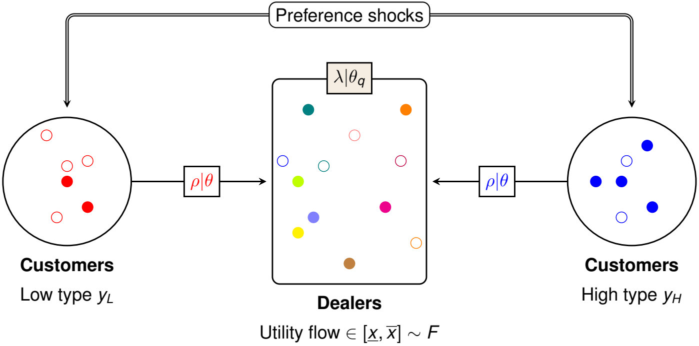
Endogenous density of trading volume



Conclusion

- Endogenous core-periphery network!
- But identities of core and periphery change over time
- Can freeze identities by setting $\gamma = 0$
- But then the steady state involves no trading
- Can fix this problem by
  - Viewing investors with $\gamma = 0$ as dealers
  - Introducing customers with time varying types who trade through dealers as in the SC market model
- Delivers a model of *Frictional Intermediation*

Frictional intermediation



1. Fully tractable setup
2. Closed forms for counterparts of key statistics
3. Calibrate to trade-level data from the municipal bonds market

References

1. Search
   - DGP (05|07), Vayanos and Weill (08), Lagos and Rocheteau (09), Feldhutter (11), Gavazza (11), Afonso and Lagos (13), Neklyudov (14), Lester-Rocheteau-Weill (15), Üslü (16), Farboodi-Jarosch-Shimer (17), HLW (19,21)
2. Empirics
   - **Chains**: Ashcraft and Duffie (07), LS (18) and HSN (18)
   - **Trading delays and price dispersion**: AD, Gavazza (11), Jankowitsch et al. (11)
   - **Core-periphery structure**: AD, Atalay and Bech (10), LS, AL, Craig and von Peter (14)
3. Network and hybrid models
   - Atkeson, Eisfeldt and Weill (15), Colliard and Demange (17)
   - Zawadowski (13), Gofman (14), Babus and Kondor (17), Malamud and Rostek (17), Glode and Opp (16), Farboodi (17)

Environment

- Measures 1 of customers and m of dealers
- Homogenous discount rate $r > 0$
- Asset supply $s \in (m, 1)$
- Agents can hold zero or one unit
- Customers get utility flow $y \in \{y_L, y_H\}$
  - Preference shocks arrive at rate γ
  - Conditional on a shock customer type is set to y_j with probability π_j
- Dealers get utility flow $x \in [\underline{x}, \overline{x}]$
- *Continuous* cross-sectional distribution F of dealer types
- Focus on *steady state*

Trading

- Customer trading:
  - CD|DC contact **rate** ρ
  - Information need not be incomplete
  - Nash bargaining with dealer bargaining power $\theta \in (0, 1)$
- Frictional interdealer market:
  - DD contact **rate** λ
  - Assume complete information
  - Nash bargaining with seller bargaining power $\theta_1 \in (0, 1)$

$\Rightarrow$ Type distributions

  - Dealers (CDF): $\Phi_0(x)$ and $\Phi_1(x)$
  - Customers (masses): μ_{L0}, μ_{H0}, μ_{L1}, and μ_{H1}

Reservation values

- Denote by $W(y)$ and $V(x)$ the reservation values
- On the *customer* side:

$$
\begin{aligned}
rW(x) &= y + \gamma \sum_i \pi_i \left(W(y_i) - W(y)\right) \\
\text{(sell2D)} \quad & + \rho\,(1-\theta) \int_{\underline{x}}^{\overline{x}} \left(V(x) - W(y)\right)^+ d\Phi_0(x) \\
\text{(buyfromD)} \quad & - \rho\,(1-\theta) \int_{\underline{x}}^{\overline{x}} \left(W(y) - V(x)\right)^+ d\Phi_1(x)
\end{aligned}
$$

- Valid on $[y_L, y_H]$ not just at y_L and y_H
- Double feedback from distributions of types and RVs

Reservation values

- On the *dealer* side:

$$rV(x) = x + \lambda \int_{\underline{x}}^{\overline{x}} \theta_1 \left(V(x') - V(x)\right)^+ \frac{d\Phi_0(x')}{m}$$

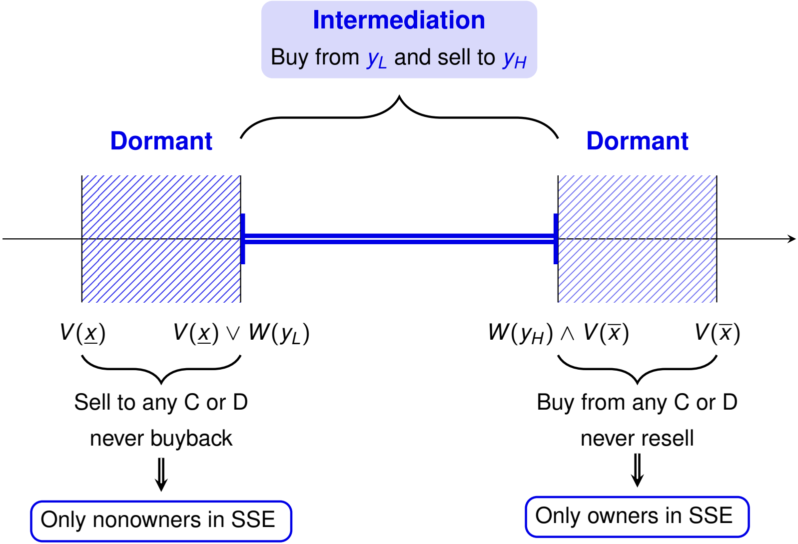
$$\text{(buyfromD)} \quad - \lambda \int_{\underline{x}}^{\overline{x}} \theta_0 \left(V(x) - V(x')\right)^+ \frac{d\Phi_1(x')}{m}$$

$$\text{(sell2C)} \quad + \rho \sum_i \theta \left(W(y_i) - V(x)\right)^+ \mu_{i0}$$

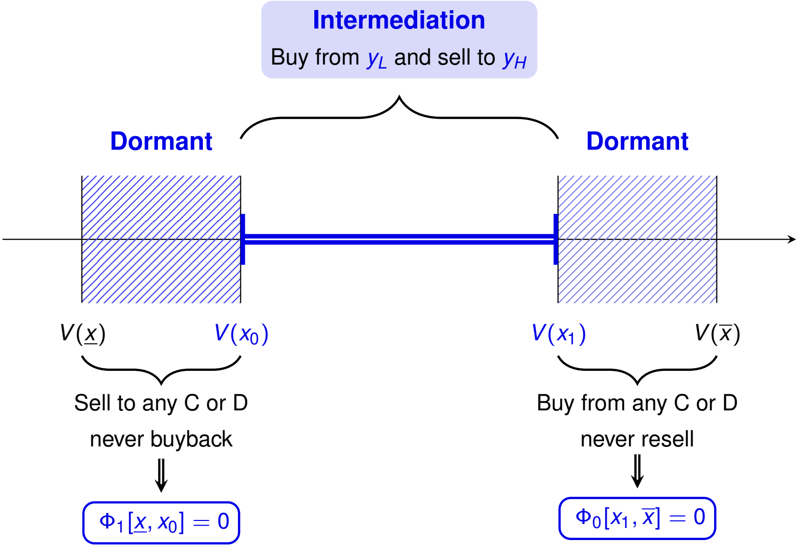
$$\text{(buyfromC)} \quad - \rho \sum_i \theta \left(V(x) - W(y_i)\right)^+ \mu_{i1}$$

- Dealer receive no preference shocks
- **Key result:** [RV$_c$–RV$_d$] admits a unique solution that is bounded, Lipschitz, and strictly increasing

Trading patterns



Trading patterns



Distributions

- Given $x_0 \leq x_1$ the distributions of types solve

$$s = \mu_{L1} + \mu_{H1} + \Phi_1(\overline{x})$$

$$0 = \pi_i - (\mu_{i0} + \mu_{i1}) = mF(x) - (\Phi_1(x) + \Phi_0(x))$$

$$0 = \Phi_1[\underline{x}, x_0] = \Phi_0[x_1, \overline{x}]$$

$$0 = \gamma \pi_L \mu_1 - \gamma \mu_{L1} - \rho \mu_{L1} \Phi_0[x_0, x_1]$$

$$0 = \gamma \pi_H \mu_0 - \gamma \mu_{H0} - \rho \mu_{H0} \Phi_1[x_0, x_1]$$

$$0 = \rho \mu_{L1} \Phi_0[x_0, \bullet] - \rho \mu_{H0} \Phi_1 - \frac{\lambda}{m} \Phi_1 \Phi_0[\bullet, x_1] \quad \text{on } [x_0, x_1]$$

- The existence of an equilibrium reduces to as a fixed point problem over the pair of constants (x_0, x_1)!

Equilibrium

- Fixed point problem:

$$
\begin{aligned}
\mathbf{x} = (x_0, x_1) &\implies \big(\mu_{jq}(\mathbf{x}), \Phi_q(x;\mathbf{x})\big) \\
&\implies (V(x;\mathbf{x}), W(y;\mathbf{x})) \\
&\implies \hat{x}_1 := \inf\{x : V(x;\mathbf{x}) \geq W(y_H, \mathbf{x})\} \wedge \overline{x} \\
&\implies \hat{x}_0 := \sup\{x : V(x;\mathbf{x}) \leq W(y_L, \mathbf{x})\} \vee \underline{x}
\end{aligned}
$$

- SSE exists (Brouwer)
- N&S Conditions for intermediation (large θ, ρ, and Δautarky)
- S Conditions for *no dormant dealers* ($\Rightarrow$ uniqueness)
- SSE with intermediation is observationaly equivalent to one w/o dormant dealers but with endogenous *s* and *m*

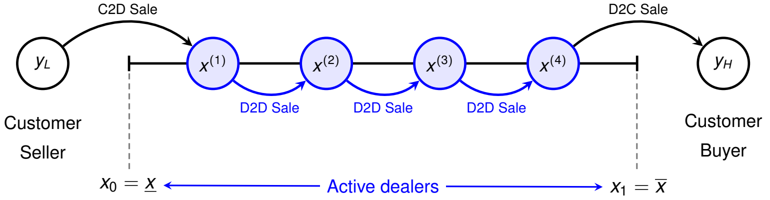
Trading patterns

- Let $m_q := \Phi_q(\overline{x})$

1. Customers
   - at y_L sell to dealers with intensity ρm_0
   - at y_H buy from dealers with intensity ρm_1
   - others do not trade
2. Dealers at x
   - buy from customers at y_L with intensity $\rho \mu_{L1}$
   - buy from dealers at $x - \Delta$ with intensity $\lambda_0(x) = \lambda \Phi_1(x)/m$
   - Sell to dealers at $x + \Delta$ with intensity $\lambda_1(x) = \lambda(m_0 - \Phi_0(x))/m$
   - Sell to customers at y_H with intensity $\rho \mu_{H0}$

$\Rightarrow$ *Endogenous* intermediation **chains**

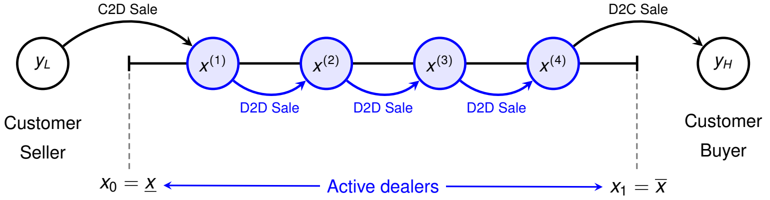
Intermediation chains



Length	Freq. (%)	Markup (%)	Share of Markup (%)				
1	77	1.85	100	·	·	·	·
2	13	1.94	43	57	·	·	·
3	7	2.26	29	23	48	·	·
4	**1**	**2.92**	**22**	**21**	**19**	**38**	·
5	0.3	3.26	19	9	25	12	35

Source: U.S. Municipal bonds, Li and Schürhoff (2018)

Intermediation chains



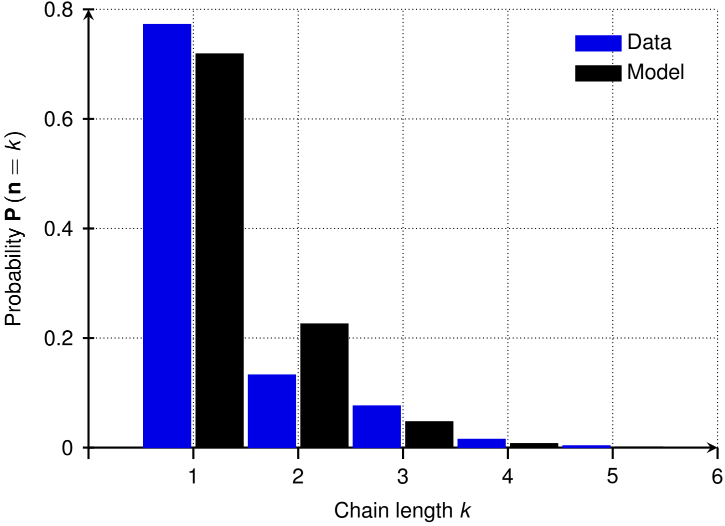
- Key statistic:

$$\mathbf{P}\left(\{\mathbf{n}=k\}\bigcap_{i=1}^{k}\left\{x^{(k)}\leq z_k\right\}\right)=\frac{1}{\chi}\prod_{i=1}^{k}\log\left(\frac{1}{\rho\mu_{H0}+\lambda_1(z_i)}\right)$$

  with the constant $\chi := \lambda_1(x_0)/\rho\mu_{H0}$

- $\mathbf{n}$ is a truncated Poisson variable with rate χ
- Municipal bond market: $\mathbb{E}[\mathbf{n}] = 1.34$ implies $\chi = 0.8618$

Municipal bond market



Calibration

- Exact identification of s (supply), m (dealers), γ, π_H, ρ, and λ
- Calibrate the rest to the average markup, the yield spread, and the beta of markup on chain length
- Along a chain:

$$\text{bid} = \theta W(y_L) + (1-\theta) V\left(x^{(1)}\right)$$

$$\text{ask} = (1-\theta) V\left(x^{(\mathbf{n})}\right) + \theta W(y_H)$$

- The model requires a high θ to match the markup level
- But then the Diamond paradox kicks in: neither bid $\simeq W(y_L)$ nor ask $\simeq W(y_H)$ depend on the dealer types

Demographic targets

- Six parameters: s, m, γ, π_H, ρ, λ
- Targets from the municipal bonds market (GHS07|LS18)
  1. Supply per capita: 0.2058

     Estimate uses trade size, supply, and participation
  2. Average chain length: 1.34

     Identifies the ratio $\chi = (\lambda m_0/m)/(\rho\mu_{H0})$
  3. Average inventory duration: 3.3 days

     Identifies the selling intensities $\rho\mu_{H0}$ and $\lambda m_0/m$
  4. D2C Turnover: 41.1%/Year

     Identifies m_1 and the product $\gamma\pi_H$
  5. Average time for a customer to sell: $\rho m_0 = 5$ days
  6. High type customers are marginal: $\pi_H = s$

Calibration

Supply per customer capita	s	0.2058	
Relative size of the dealer sector	m	0.0041	
Type switching intensity	γ	0.5267	
Probability of a switch to high	π_H	0.2058	
Intensity of customer-to-dealer contact	ρm	76.87	
Intensity of dealer-to-dealer contact	λ	78.04	
Customer: time to contact dealer owner	$1/(\rho m_1)$	9.303	days
Customer: time to contact dealer non owner	$1/(\rho m_0)$	5.000	days
Dealer: time to contact H0 customer	$1/(\rho \mu_{H0})$	4.303	days
Dealer: time to contact L1 customer	$1/(\rho \mu_{L1})$	8.007	days
Dealer: time to contact m0 dealer	$m/(\lambda m_0)$	4.925	days
Dealer: time to contact m1 dealer	$m/(\lambda m_1)$	9.164	days
Assets held in dealer sector	m_1/s	0.71%	1%

Assortative matching

- Heterogeneity among high types customers: $y_H + z$ with extra flow z drawn from some G upon switching to high type
- Dealers indexed by $x \in [\underline{x}, \overline{x}]$
  - High x dealers match with high z customers
  - Homogenous utility type y_L
- Dealers at x only sell to $y_H + z_x$ with $m_1 F(z_x) = \Phi_1(x)$
- Same trading patterns as in benchmark!
- But $V(x)$ is now much steeper due to the higher flow valuation of customer buyers

$\Rightarrow$ No longer require a high bargaining power to match the observed dependence of markups on chain length

Markup splits

Not targeted in the calibration process

	Extended model							Data						
n	1	2	3	4	5	6	7	1	2	3	4	5	6	7
1	100	·	·	·	·	·	·	100	·	·	·	·	·	·
2	54	46	·	·	·	·	·	43	57	·	·	·	·	·
3	46	10	44	·	·	·	·	29	23	48	·	·	·	·
4	42	8	8	42	·	·	·	22	21	19	39	·	·	·
5	39	6	6	6	41	·	·	19	9	25	12	34	·	·
6	37	5	5	5	5	43	·	17	8	13	24	8	32	·
7	35	5	5	5	5	5	40	17	6	12	14	12	8	31

Next time

- Back to a semicentralized market setting but now with incomplete information about investor types
- Myerson-Satherwaite (88): Impossibility theorem
- Two alternative price-setting mechanisms
  - Screening by dealers (TIOLI)
  - Directed (aka competitive) search
- Open problems